THEORETICAL AND EXPERIMENTAL STRESS ANALYSIS OF A COMPOSITE PRESSURE VESSEL

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Abstract

The stress and failure analysis of a composite pressure vessel is presented. The vessel is composed of hoop and angle windings of glass/epoxy. One end of the vessel has a chopped fiber composite cap and the other a metal cap. Two lay-up configurations of the vessel are analyzed for internal pressure loading. Stress analysis is done by using a finite element computer program for axisymmetric bodies with solid elements. The results show the existence of stress concentration in regions where the windings are bonded to the end caps. Failure of the vessel is predicted by applying Hill's criterion and a hybrid type criterion (Ref. 7), to the stresses in each layer. The effect of increased strength due to cross-overs of the windings is discussed. It is felt that this effect requires further investigation. The behavior of the vessel after cracking of the matrix is also studied. Results of pressure tests of the vessel are presented, showing reasonably good agreement with analytical predictions.

I. Introduction

In designing a composite structure one usually tries to have it fail in a fiber breakage mode rather than by matrix cracking and separation of fibers and layers. It is shown that for a pressure vessel which is a "one-shot" type structure, this approach may lead to an over-conservative design. Instead, it is possible to allow the matrix to fail first if an elastomeric layer is inserted on the internal surface of the vessel. This layer makes it possible for the vessel to maintain pressure even when the matrix is cracked. When such a layer is applied it is possible to have a very light structure which will function satisfactorily even though it is partially inelastic.

The work which is described in this paper deals with a cylindrical pressure vessel of S-glass/epoxy filament winding. One end of the cylinder has a cylindrical steel insert and the other end is closed by a dome which is made of glass mat in phenolic resin.

Material properties of the vessel are calculated from the basic engineering constants of a uniaxial ply and its constituents. Certain assumptions had to be made in order to get some of the constants (such as ν_{23}) for which no data was available. Stress analysis of the vessel is done by the finite element method using a program for axisymmetric bodies with solid ring elements (Ref. 1). Failure is determined by applying Hill's criterion (Ref. 2) and a hybrid criterion (Ref. 7) to each layer. The effect of increased lateral and shear strength due to cross-overs was studied in Ref. 3. The effect is taken into account in this work and a correction factor is determined.

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II. Description of Structures

A general description of the vessel is given in Figure 1 below.

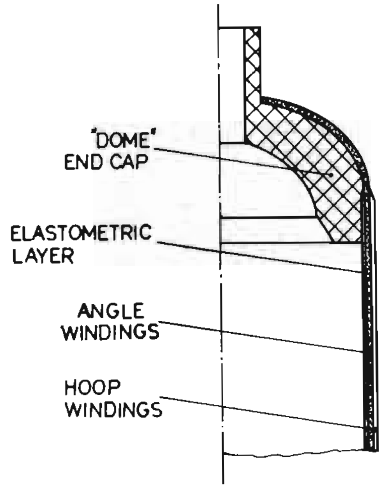

FIG.1. CROSS SECTION OF THE PRESSURE VESSEL

The material and layup configuration of the windings are summed up below for the two lay-up configurations considered.

Configuration	Winding* Angle	No. of Windings	Thickness mm.	Material
1	90°	2	Total .3	S-901 Glass Epoxy
	±24°	1	Total .7	
2	90°	26	Total 2.6	Same
	±24°	4	Total 2.8	

* Measured from cylinder axis

The engineering modulii of the constituent materials are given below:

Windings*

$E_{11}, 10^6$ psi	$E_{22}, 10^6$ psi	$G_{12}, 10^6$ psi	ν_{12}
7.5	3.0	1.2	0.25

* Basic properties of a uniaxial lamina.

"Dome" End Cap

$E, 10^6$ psi	$G, 10^6$ psi	ν
2.8	1.05	0.33

Steel Insert

$E, 10^6$ psi	$G, 10^6$ psi	ν
30.0	11.5	0.3

III. Stress Analysis

Determination of Material Constants

The analysis assumes that the windings can be represented by layers of orthotropic material (with nine material constant). The stress strain relations can be written as follows:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix}$$

where:

1- fiber direction

2- transverse direction

3- direction normal to 1-2 plane

Normally the kind of information one gets from tests and data sheets is in the form of engineering modulii. The relations between the engineering constants and the compliance matrix terms are given below for the material symmetry axes 1, 2, 3.

$$S_{11} = 1/E_{11}$$
$$S_{12} = -\nu_{12}/E_{11}$$
$$S_{22} = 1/E_{22}$$

If we assume transverse isotropy (in the 2-3 plane) we get:

$$S_{13} = S_{12} = -\nu_{12}/E_{11}$$
$$S_{33} = S_{22} = 1/E_{22}$$
$$S_{44} = 2(S_{22} - S_{23})$$
$$S_{55} = S_{66} = 1/G_{12}$$
$$S_{23} = -\nu_{23}/E_{22}$$

Since the value for ν_{23} was not available it was taken as an average:

$$\nu_{23} = (\nu_f + \nu_m)/2 \qquad \begin{matrix} f - \text{fiber} \\ m - \text{matrix} \end{matrix}$$

so that:

$$S_{23} = -(\nu_f + \nu_m)/2E_{22}$$

In order to get the coefficients in the cylindrical coordinate system, the matrix has to be transformed by the appropriate transformation matrix (Ref. 4). The transformed matrix can then be inverted in order to get the material coefficient matrix [C] in the cylindrical coordinates of the body.

The Finite Element Analysis

The stress analysis of the vessel is done by using a finite element computer program which is based on Ref. 1. The program can accommodate axisymmetrical bodies with asymmetric loading and with orthotropic materials. The elements which are used in the program are solid ring element.

For purpose of analysis, the structure was divided into two parts, each of which was treated separately. One was the dome region and the other the region around the steel insert. The results away from the ends converge to the solution of the middle section of the cylinder and were used to analyze that region. The dome region was found to be more important in the analysis and is described more extensively. A description of the finite element model of the dome region is given in Fig. 2. The figure is distorted out of scale for clarity purposes. The windings are treated as two different materials, one having the properties of a laminate of $\pm 24^{\circ}$ lay-up and the other is composed of the hoop windings (Materials Nos. 1 and 2, respectively, in Fig. 2)

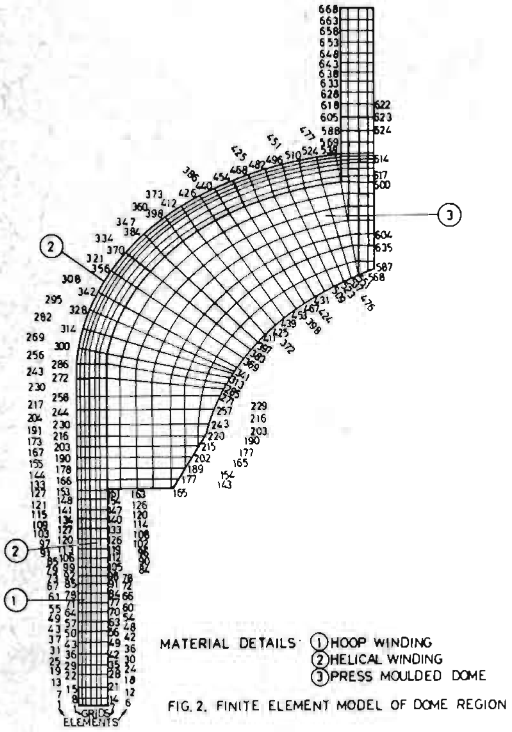

FIG. 2. FINITE ELEMENT MODEL OF DOME REGION

The pressure used in the analysis was 100 Kg/cm^2. Figs. 3 and 4 show the variation of the stresses with distance along the axis in the hoop and angle windings respectively. Fig. 5 gives the stresses in the dome along the interface with the windings.

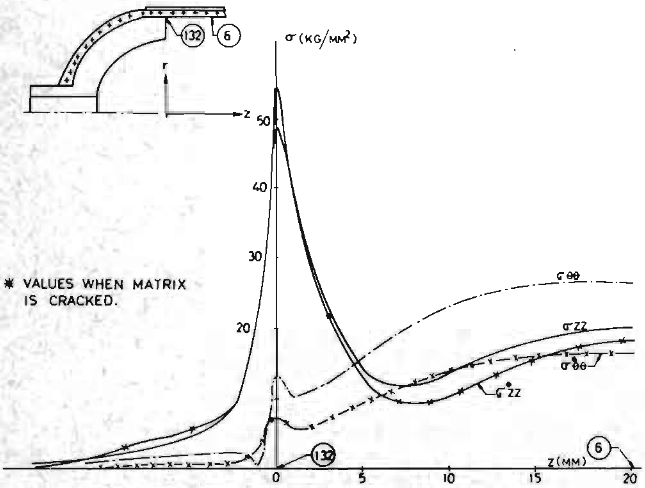

FIG. 3.-CALCULATED STRESSES IN THE ANGLE-WOUND LAYER.

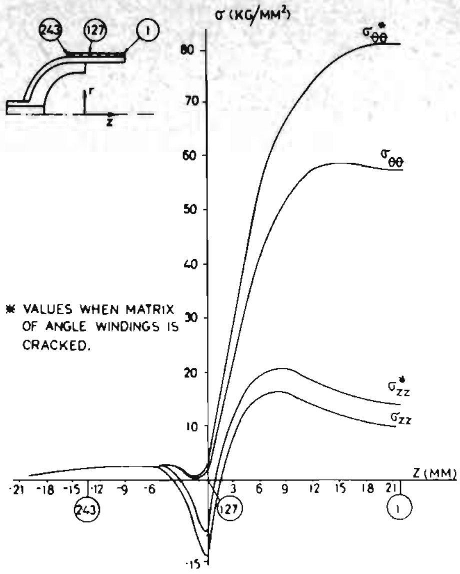

FIG.4.-CALCULATED STRESSES IN THE HOOP-WOUND LAYER.

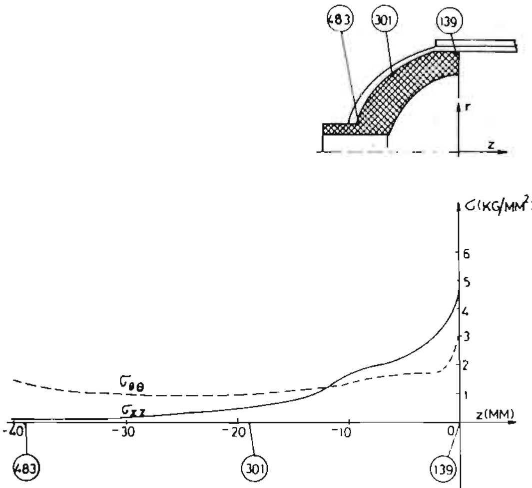

FIG. 5. THE CALCULATED STRESSES IN THE MOLDED DOME AT THE INTERFACE WITH THE HELICAL WINDING.

IV. Failure Analysis

There have been various attempts to develop failure criteria for composite materials under combined stresses. No satisfactory generalized criterion has been found to date for a laminate as a whole. There are, however, several criteria for failure of the individual layers in a conventional laminate, where the layers are merely stacked and there is no weaving or cross-over effect which interconnect the layers.One of the criteria which is widely used is Hill's criterion. The criterion was originally developed for metals but adapted for orthotropic composite layers by Tsai (Ref. 2).

A more recent criterion, which was proposed by Hashin (Ref. 7), expresses the two distinct failure modes of a lamina, namely fiber failure and matrix cracking. The above mentioned criteria are given by the following mathematical expressions:

Hill's:

$$\left(\frac{\sigma_{11}}{\sigma_{11F}}\right)^2 + \frac{1}{r}\left(\frac{\sigma_{11}}{\sigma_{11F}}\right)\times\left(\frac{\sigma_{22}}{\sigma_{22F}}\right) + \left(\frac{\sigma_{22}}{\sigma_{22F}}\right)^2 + \left(\frac{\sigma_{12}}{\sigma_{12F}}\right)^2 = 1$$

where:

$$\frac{1}{r} = \sigma_{22F}/\sigma_{11F}$$

$\sigma_{11F}, \sigma_{22F}, \sigma_{12F}$ — Basic material strengths in fiber direction, transverse direction and shear

Hashin's:

either $\sigma_{11} = \sigma_{11F}$ (fiber failure)

or $(\sigma_{22}/\sigma_{22F})^2 + (\sigma_{12}/\sigma_{12F})^2 = 1$ (matrix failure)

Both of these criteria were used for prediction of cracking of the matrix and the results came out very close to one another. For fiber failure prediction, the second criterion alone was used.

The analysis shows that failure will first occur in the matrix of the angle windings. The subsequent propogation of failure in the structure is studied by analyzing a new, effective structure with degraded properties for the cracked material. Thus, step-wise changes in material properties were employed following each prediction of new matrix cracking in a particular set of layers. This procedure can be applied in the present analysis because of the relative simplicity of the structure and loading. Results show that as the load increases, failure spreads from the matrix of the angle windings to that of the hoop windings and finally a complete collapse occurs, corresponding to failure of the fibers in the hoop direction. This mechanism was confirmed by observation during the pressure tests and inspection of the slope of the pressure-strain curve.

Fig. 6 shows the experimental pressure-strain curve and three distinct regions are observed. At low pressure, the cylinder is intact and the curve is linear. At a certain pressure, the matrix starts to fail and the slope of the curve changes until a new slope is reached which corresponds to the situation where all the layers carry loads in the fibers alone.

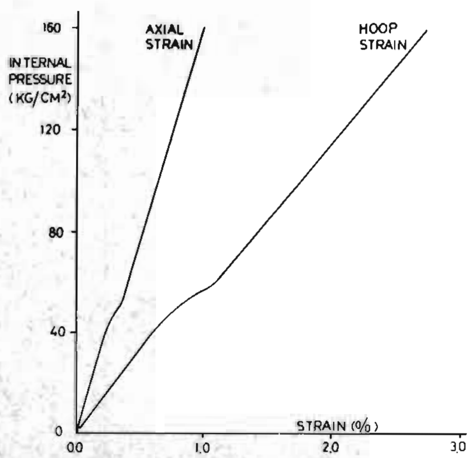

FIG.6. TYPICAL PRESSURE STRAIN CURVES OF A VESSEL UNDER INTERNAL PRESSURE.

Fig. 7 shows a picture of a vessel after complete failure. It can be seen that, in the angle winding layers, only the matrix is failed, while in the hoop winding layers, both the matrix and fibers are failed.

FIG.7.-A TYPICAL FAILURE OF A VESSEL UNDER INTERNAL PRESSURE

Failure Analysis Procedure

The procedure used is as follows. Stress output for each element, which may represent a combination of several layers, is obtained from the finite element analysis. This is then used to get all the stresses of a particular layer in the element. For example, if we have the stress results for an element of $\pm 24^\circ$ material, the following equation is used to get the stresses in either the $+24^\circ$ or -24° layer:

$$\{\sigma\}_k = [C]_k\,[\bar{C}]^{-1}\,\{\bar{\sigma}\}$$

where:

$[C]_k, \{\sigma\}_k$ - material matrix and stresses of the k^{th} layer

$[\bar{C}], \{\bar{\sigma}\}$ - material matrix and stresses of the element ($\pm 24^\circ$, combined)

The above equation gives the stresses of a layer referred to the (r, θ, z) coordinate system of the cylinder. These stresses are then transformed to the layer's symmetry axes 1, 2, to give $\sigma_{11}, \sigma_{22}, \sigma_{12}$. The calculated values of $\sigma_{11}, \sigma_{22}, \sigma_{12}$ are then used in the failure criteria to predict failure initiation. Stress components in the thickness direction 3 are found to be unimportant in the thin shell problem under consideration. More details on this procedure can be found in Ref. 2, 3 and 6.

Strength of a Unidirectional Layer and the Effect of Winding Crossovers

The strength values which are normally used in Hill's and Hashin's criteria are those of a unidirectional lamina. These values seem to be adequate for σ_{11F} but much too low for σ_{22F} and σ_{12F} in the case of a helically wound vessel. This strengthening effect for transverse and shear stresses, is due to the fact that the fibers cross over one another rather then lying in stacked layers as in the case in an ordinary laminate. The effect is sometimes called "crossover effect" and it is discussed extensively in Ref. 3.

A picture of crossover pattern can be seen in Fig. 8 which shows the vessel during winding. The amount of strengthening of the material for transverse and shear stresses depends on the geometry of the cylinder and the winding pattern. For the particular vessel and lay-up under consideration, the experimental results in Ref. 3 indicate that the transverse and shear strengths are 3 times the values which are used for a conventional laminate. These values are summed up in the table below:

Strength of Unidirectional Glass /Epoxy,kg/mm^2

Stress Component	Conventional Laminate	Vessel Under Study
σ_{11F}	190	190
σ_{22F}	8.5	25.5
σ_{12F}	7.0	21.0

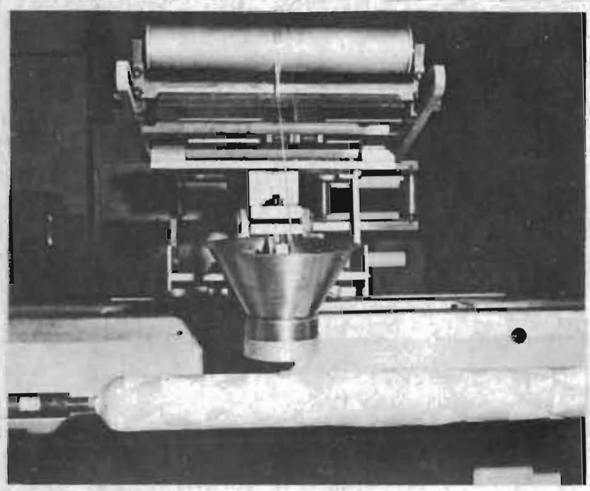

FIG.8.- WINDING OF A TYPICAL VESSEL

Results of Failure Analysis and Comparison with Pressure Tests

The analysis showed that failure of the matrix of the angle windings will start near the dome end cap at a pressure of 70 kg/cm^2 while the matrix at regions away from the ends will fail at 90 kg/cm^2. This earlier cracking of the matrix near the end cap is due to local stress concentration. Final breaking of the vessels occurs at a pressure of 210 kg/cm^2.

The experimental results showed cracking of the matrix at pressures of 50 - 90 kg/cm^2. Cracking was observed by both the sounds associated with it and the change of slope on the pressure vs. strain curve. Cracking of the matrix is normally associated with loss of pressure due to leaks. In order to be able to maintain pressure after the matrix is cracked, the inner surface of the vessel is coated with an elastomeric layer. Thus, it is possible to carry the tests up to complete failure of the vessel by fiber breaking. The value which was obtained is 230 kg/cm^2.

The analysis of configuration 2, the thicker version of the vessel, gave the following results:

matrix failure	280-350	kg/cm^2
fiber failure	1400	kg/cm^2

V. Discussions and Conclusions

The results of the analysis and tests show that failure starts by matrix cracking which causes pressure leaks. If an elastomeric layer is applied to the inner surface of the vessel no pressure leaks occur and the vessel can be loaded to complete failure (bursting), which occurs when the hoop windings break. If the vessel does not have to undergo cyclic loading, or is intended for a single use, then cracking of the matrix should not be considered as failure and the vessel should be designed for fiber failure together with means to ensure no pressure leaks. Using this approach can save material and reduce costs as is indicated by comparing configuration 2 and 1, both of which are intended for a maximum load of 100 kg/cm^2. The effect of strengthening due to crossovers of the helical windings seems to triple the lateral and shear strength of the windings as compared to conventional laminates. This effect needs further study so that the phenomenon is completely understood and strength properties of a layer in helical winding can be determined for any geometry.

The two failure criteria gave almost identical results for the initiation of matrix cracking. This was due to the fact that the stress in the fiber direction was small compared to the strength in that direction. Hashin's criterion is however believed to be more meaningful physically since it seems to agree also with the failure mechanism which was observed experimentally, i.e., distinct matrix cracking and fiber breakage modes.

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